On the Merits of Orthogonalizing Powered and Product Terms: Implications for Modeling Interactions among Latent Variables

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Abstract

The goals of this report are two-fold: (a) briefly highlight the merits of residual centering for representing interaction and powered terms in standard regression contexts (e.g., Lance, 1988) and (b) extend the residual centering procedure to represent latent variable interactions. The proposed method for representing latent variable interactions has potential advantages over extant procedures. First, the latent variable interaction is derived from the observed covariation pattern among all possible indicators of the interaction. Second, no constraints on particular estimated parameters need to be placed. Third, no recalculations of parameters are required. Fourth, model estimates are stable and interpretable. In our view, the orthogonalizing approach is technically and conceptually straightforward, can be estimated using any SEM software package, has direct practical interpretation of parameter estimates, its behavior in terms of model fit and estimated standard errors is very reasonable, and it can be readily generalized to other types of latent variables where nonlinearity or collinearity are involved (e.g., powered variables).
On the Merits of Orthogonalizing Powered and Product Terms: 
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Research in the social sciences frequently includes hypotheses about interactive or nonlinear effects on a given outcome variable. When it comes to estimating such effects, however, there is lack of consensus on how to do so properly, particularly when performing structural equation modeling (SEM). A plethora of methods have been proposed and discussed, including those described in Algina and Moulder (2001), Jaccard and Wan (1995), Klein and Moosbrugger (2000), Klein and Muthén (2002), Marsh, Wen, and Hau (2004), Ping (1996a, 1996b), Schumacker and Marcoulides (1998), and Wall and Amemiya (2001, 2003). Most approaches to latent variable interactions are based on a product-indicator methodology originated by Kenny and Judd (1984) that requires a level of technical and computational sophistication that render them quite inaccessible to the average practitioner.

The focus of this report is on the development of an intuitive and technically straightforward approach to estimating interactive and nonlinear effects within SEM. To clarify our position, however, we will first review methods for estimating nonlinear effects within multiple regression, and discuss some of the difficulties inherent within the regression framework. Our focus will be on the use of residual centering (Lance, 1988) versus mean centering (Cohen, 1978; Cronbach, 1987). We will then present an extension of residual centering (which we refer to as ‘orthogonalizing’) to represent nonlinear interaction effects in latent variable models.

Nonlinear Effects in Multiple Regression

Although our ultimate goal is to explicate the usefulness of residual centering in the context of latent variable analyses, we must first highlight some of the merits of residual centering interaction and powered terms in standard multiple regression contexts. In ordinary
least-squares (OLS) regression, the product of two variables can be used to represent the interactive effect, as seen in equation 1:

\[ Y = b_1X + b_2Z + b_3XZ + e \]  

where \( Y \) is the outcome variable of interest, \( e \) is the assumed error term, \( X \) and \( Z \) are the first-order predictor variables, and \( XZ \) is the multiplicative or product term that represents the interaction effect. Essentially, this regression equation specifies that the slope of the line relating \( X \) to \( Y \) changes at different levels of \( Z \), or equivalently, that the slope of the line relating \( Z \) to \( Y \) changes at different levels of \( X \). Saunders (1956) first demonstrated that a product term accurately reflects a continuous variable interaction. Similarly, natural polynomial, or “powered”, variables (\( X^2, X^3, \) etc.) can be used to represent higher-order nonlinear effects of a variable such as a quadratic or cubic trend of age or time.

Researchers often struggle with the fundamental problem that the product term may be highly correlated with the first-order predictor variables from which it is derived, and that a powered term is similarly highly correlated with the original predictor variable from which it is derived. When predictor variables are correlated, the collinearity can lead to problems when estimating regression coefficients. Collinearity means that within the predictor set, one or more of the independent variables (IVs) are highly predicted by one or more of the other IVs. The collinear variables have regression coefficients that are poorly estimated and minor fluctuations in the sample, such as those caused by measurement and sampling error, have major impacts on the weights. The inherent collinearity of powered and product terms with their first-order predictor variables is problematic because it can create instability in the values for the estimated regression weights, leading to ‘bouncing beta weights’ (Pedhazur, 1982).

Ideally, an interaction term is uncorrelated with (orthogonal to) its first-order effect
terms. When modeling the relationship between a continuous outcome variable and a categorical predictor variable or a continuous variable that has been measured discretely with a finite range, contrast coding may be used to estimate interactive effects that are either uncorrelated (when \( n \)’s are equal) or minimally correlated (when \( n \)’s are unequal) with first-order effects. Under orthogonal conditions, when the interaction term is entered into a model, the partial regression coefficients representing the magnitudes, directions, and significances of the main effect variables remain precisely the same as they were before the interaction was included. For powered terms, orthogonal polynomial contrast codes are available, but there is a practical limitation that the predictor variable must be measured discretely with a very limited range of values so that tables of contrast coefficients are readily available (see Cohen, Cohen, West, & Aiken, 2003, p. 215, for example). With continuous variable interaction terms, the orthogonality property is harder to achieve. Several sources (i.e., Aiken & West, 1991; Cohen, 1978; Cronbach, 1987) have demonstrated that if the first-order effect variables are transformed from a raw-score scaling to a deviation-score scaling by subtracting the variable mean from all observations (i.e., mean centering) the resulting product term will be minimally correlated or uncorrelated with the first-order variables depending on the proximity to bivariate normality.

Despite the equivalent partial regression coefficients representing the relative contributions of un-centered versus mean-centered first-order variables to the regression equation (see Kromrey & Foster-Johnson, 1998, for a convincing demonstration), mean centering predictor variables prior to creating interaction or product terms has two distinct advantages. First, mean centering alleviates the ill-conditioning of the correlation matrix among the predictor variables that results from *non-essential multicollinearity* (Marquardt, 1980) among the first-order predictors and their interaction term (or between first-order predictors and even powered
terms such as between $X$ and $X^2$, $X^4$, or $X^6$). Thus, the resultant instability of regression estimates and standard errors are stable and robust (i.e., the “bouncing beta weight” problem is remedied).

The second advantage of mean centering concerns the interpretability of the estimates. The regression coefficient for a mean-centered predictor may be more practically meaningful than the same coefficient for the same predictor with an arbitrary zero point (i.e., interpreting the relative size of change in $Y$ for a one-unit change in $X$ at a given level of $Z$ may be easier if the zero point of $Z$ is the average value of $Z$ rather than an arbitrary and non-meaningful scale value). Interpretability may also be improved by plotting the predicted relationship between $X$ and $Y$ over a range of plausible $z$-scores (e.g., Aiken & West, 1991; Cohen et al., 2003; Mossholder, Kemery, & Bedeian, 1990).

Under most circumstances, mean centering is an adequate solution to the collinearity problem. At times, however, the mean-centered product or powered term will still have some degree of correlation with its first-order variables that can influence the partial regression coefficients. To remedy this lack of complete orthogonality with the mean-centering approach, a simple two-step regression technique called residual centering can be used that ensures full orthogonality between a product term and its first-order effects (Lance, 1988). This technique is also generalizable to powered terms.

Residual centering (i.e., orthogonalizing) is a comparable alternative approach to mean centering that also serves to eliminate non-essential multicollinearity in regression analyses. Residual centering, as originally suggested by Lance (1988), is essentially a two-stage ordinary least squares (OLS) procedure in which a product term or powered term is regressed onto its respective first-order effect(s). The residuals of this regression are then used to represent the
interaction or powered effect. The variance of this new orthogonalized interaction term contains the unique variance that fully represents the interaction effect, independent of the first-order effect variance (as well as general error or unreliability). Similarly, the variance of an orthogonalized powered term contains the unique variance accounted for by the curvature component of a nonlinear relationship, independent of the linear components.

Residual centering, like mean centering, has a number of inherent advantages for regression analyses. First, the regression coefficients for orthogonalized product or powered terms are stable. That is, the regression coefficients and standard errors of the first-order effect terms remain unchanged when the higher-order term is entered. Second, the significance of the product or powered term is unbiased by the orthogonalizing process. Third, unlike mean centering, orthogonalizing via residual centering ensures full independence between the product or powered term and its constituent main-effects.¹

A Simple Regression Example

For this example, we use two concepts from the area of perceived control and their interaction to predict well-being. The first concept, agency for ability, is the degree of belief that one is smart and intellectually able. The second concept is one’s degree of belief that successful intellectual performance comes about because of unknown causes. In this example, ample research supports the hypothesis that agency for ability is positively related to well-being or positive affect (for a review, see Skinner, 1996). Depending on the degree to which one believes that unknown factors are also responsible for successful performance, this positive relationship would be moderated. The reason for this expected interaction is simply that the more one believes unknown factors are responsible for performance, the more that the well-being-enhancing effects of agency would be undermined. Specifically, we expected that high beliefs in
unknown causes would reduce the otherwise positive relationship of agency for ability with positive affect.

To test this hypothesis and to compare methods, two product terms for the interaction between agency for ability (Agency) and one’s belief in unknown causes (Causes) were created:

\[ INT_U = \text{Agency} * \text{Causes} \]  \hspace{1cm} (2)

\[ INT_{MC} = (\text{Agency} – \text{Mean of Agency}) * (\text{Causes} – \text{Mean of Causes}) \]  \hspace{1cm} (3)

The interaction term in equation 2, \( INT_U \), is the product of the uncentered first-order effect variables. The interaction term in equation 3, \( INT_{MC} \), is the product of the mean-centered first-order effect variables. To create a third orthogonalized interaction term, \( INT_O \), we regress \( INT_U \) on both of the untransformed first-order effect variables (Agency & Causes), and save the residuals as a new variable (\( INT_O \)).

Table 1 shows the correlations among the outcome variable of Positive Affect, the main effect variables of Agency and Causes, and the three alternative specifications of the interaction term. Table 2 shows the estimated parameters from the three alternative interaction model specifications. Although the correlation between the uncentered interaction term (\( INT_U \)) and the first-order effects of Agency and Causes is at a modest to high level (.37 and .80, respectively), the effect of this level of multicollinearity is pronounced, as shown in Table 2. The regression coefficients for the first-order effects became highly inflated in magnitude when \( INT_U \) was entered into the model. For the mean-centered term (\( INT_{MC} \)) and the orthogonalized term (\( INT_O \)), the regression coefficients for the first-order effects remained quite stable, with one notable exception. Namely, with the inclusion of \( INT_{MC} \), the regression coefficient for Causes showed a very slight drop from .05 to .04. This drop is sufficient to change the significance of the effect. Using a one-tailed test (because we know from ample research the direction of the Causes
effect), Causes is a significant predictor of Positive Affect at the $p < .05$ level in the first-order effects only model as well as in the model when the orthogonalized term (INTO) is entered. When the mean-centered term is entered (INTMC), Causes fails to reach significance at the $p < .05$ criterion. Although the correlation of INTMC with Causes was only .06, this degree of collinearity is sufficient to influence a decision about the significance of the first-order effect term. Thus, even this simple example demonstrates that there remains some influence on the regression coefficients between mean-centered predictors and their interaction term when the mean-centered term is not completely orthogonal.

A Simple Extension to Represent a Latent Variable Interaction

Although mean centering and residual centering are beneficial to the interpretation of interactions in regression models, the estimation of interaction effects within regression models is still fraught with difficulty. Perhaps the largest concern is the effect of measurement error on the power to detect such effects. OLS regression assumes that all variables are measured without error, or are perfectly reliable, an assumption that is often not tenable. Violations of this assumption may result in unknown bias in the parameter estimates (Busemeyer & Jones, 1983). Although the presence of measurement error is problematic for all variables in regression, it is particularly troublesome for an interactive or nonlinear term, for which the reliability is a function of the reliability of its constituent variables. The resultant reliability for the product term is often lower than the minimum reliability of either of the first-order effects. A second, related concern is the differentiation of multiplicative and nonlinear effects under such conditions of low power. A complete discussion of this problem is beyond the scope of this article; for a more thorough treatments see Cortina (1993), Ganzach (1997), Kromrey and Foster-Johnson (1998), Lubinski and Humphreys (1990), and MacCallum and Mar (1995).
Structural equation modeling (SEM) represents an important advance in the study of multiplicative or nonlinear effects because of its ability to address properly the presence of measurement error within a statistical model. In SEM, the proportion of variance common to multiple indicators of a given construct is estimated and the structural relations among latent constructs may then be estimated such that the relationships are estimated without the attenuating effects of measurement error. Numerous authors have described techniques to represent latent variable interactions within the context of SEM. Most approaches are based on the Kenny and Judd (1984) product-indicator model and require complex nonlinear constraints (see e.g., Algina & Moulder, 2001; Jaccard & Wan, 1995; Jöreskog & Yang, 1996; Schumacker & Marcoulides, 1998; Wall & Amemiya, 2001). Bollen (1995, 1996) and Bollen and Paxton (1998) presented a two-stage least squares (2SLS) approach that does not require the nonlinear constraints but has been found to be less effective than other methods (Moulder & Algina, 2002; Schermelleh-Engel, Klein, & Mooresbrugger, 1998). Klein and Mooresbrugger (2000) proposed a latent moderated structural model approach (LMS) utilizing finite mixtures of normal distributions which was further refined by Klein and Muthén (2002) as a quasi-maximum likelihood (QML) approach. The QML approach was found to perform well under conditions where first-order indicators are normally distributed (Marsh et al., 2004). Finally, Marsh et al. (2004) proposed an unconstrained product-indicator approach, which they report performed well, especially when underlying distributional assumptions are not met.

All major SEM software programs can implement the nonlinear constraints that are necessary to implement the procedures based on the Kenney and Judd (1984) product-indicator model. The 2SLS approach can be directly implemented through PRELIS, the pre-processor for LISREL (Jöreskog & Sörbom, 1996). The LMS approach is implemented directly in Mplus.
version 3 (Muthén & Muthén, 1998-2004). Despite the direct software implementation of the
2SLS and LMS approaches and the widespread availability of software to implement nonlinear
constraints, modeling nonlinear interactions and powered terms in the SEM context remains an
extremely difficult endeavor for the average practitioner. The direct implementations in PRELIS
and Mplus make latent variable interactions more accessible, but the researcher is limited to only
those two software programs. In our view, the orthogonalizing technique that we will describe
here is less technically demanding than other alternative methods of including interactive and
powered terms in latent variable models based on nonlinear constraints, it can be implemented in
any SEM software platform, and it provides reasonable estimates that are comparable to other
existing procedures.

The following sections detail the implementation of the orthogonalizing procedure and
provide limited simulation evidence as to the comparability of orthogonalizing to other recently
proposed procedures such as Marsh et al’s (2004) unconstrained approach and the LMS (Klein &
Moosbrugger, 2000) approach as implemented in Mplus. Because we are comparing the
orthogonalizing approach to the LMS approach directly implemented in Mplus, all statistical
analyses were also done in Mplus. However, any differences in statistical outcomes that may
occur when implementing the orthogonalizing approach in other software should be negligible
and attributed entirely to the particular set of programming algorithms written into the particular
software. Mplus version 3 was also used to generate and analyze all simulation data.

Extending the Empirical Example to a Latent Variable Interaction

To create orthogonalized indicators for a latent interaction construct, each possible
product term from two sets of indicators for two latent constructs is formed (much like the
Kenny and Judd, 1984, approach). Building on the regression example presented in Table 2, we
can disaggregate the agency and causes variables to create three unique indicators of each
dimension. Specifically, we have three indicators of agency (ag\textsubscript{1}-ag\textsubscript{3}) and three indicators of
unknown causes (uc\textsubscript{1}-uc\textsubscript{3}). From these indicators, nine possible product terms are possible:

\begin{align*}
\text{aguc}_{11} &= \text{ag}_1 \times \text{uc}_1 \\
\text{aguc}_{12} &= \text{ag}_1 \times \text{uc}_2 \\
\text{aguc}_{13} &= \text{ag}_1 \times \text{uc}_3 \\
\text{aguc}_{21} &= \text{ag}_2 \times \text{uc}_1 \\
\text{aguc}_{22} &= \text{ag}_2 \times \text{uc}_2 \\
\text{aguc}_{23} &= \text{ag}_2 \times \text{uc}_3 \\
\text{aguc}_{31} &= \text{ag}_3 \times \text{uc}_1 \\
\text{aguc}_{32} &= \text{ag}_3 \times \text{uc}_2 \\
\text{aguc}_{33} &= \text{ag}_3 \times \text{uc}_3
\end{align*}

Each of the resulting nine uncentered product terms is then individually regressed onto
the first-order effect indicators of the constructs. For instance,

\begin{equation}
\text{aguc}_{11} = b_0 + b_1 \text{ag}_1 + b_2 \text{ag}_2 + b_3 \text{ag}_3 + b_4 \text{uc}_1 + b_5 \text{uc}_2 + b_6 \text{uc}_3
\end{equation}

where ag\textsubscript{1-3} and uc\textsubscript{1-3} represent the first-order indicators for the constructs Agency and unknown
causes (Causes), respectively. The residual for this regression would then be saved and used as
an indicator of the interaction construct. The procedure would be repeated for each of the nine
uncentered product terms. The complete SAS code for the above data manipulation to create the
initial uncentered product terms and residual centered terms can be found in Appendix A.

The next step to representing the latent variable interaction is to include the nine
orthogonalized product terms as indicators of a single latent interaction construct. Figure 1
displays the interaction plot based on OLS regression parameter estimates reported in Table 2.
Figure 2 displays the SEM path diagram used to represent the latent variable interaction. The SEM parameterization shown in Figure 2 includes two distinct features. First, there is unique variance common to the nine indicators, depending on which first-order effect indicators were used to create them. Accordingly, correlations between the residual variances of the interaction indicators should be specified, such that the indicators aguc\(_{11}\), aguc\(_{12}\), and aguc\(_{13}\) would be allowed to have correlated residuals (as derived from equations 4-6, each contains the uniqueness of ag\(_1\)). Similarly, the indicators labeled aguc\(_{21}\), aguc\(_{22}\), and aguc\(_{23}\) should have correlated residuals (each contains the uniqueness of ag\(_2\), see equations 7-9). The same pattern of expected residual correlation would be found for aguc\(_{31}\), aguc\(_{32}\), and aguc\(_{33}\), which share ag\(_3\); aguc\(_{11}\), ageuc\(_{21}\), and aguc\(_{31}\), which share uc\(_1\); aguc\(_{12}\), aguc\(_{22}\), and aguc\(_{32}\), which share uc\(_2\); and aguc\(_{13}\), aguc\(_{23}\), and aguc\(_{33}\), which share uc\(_3\).

The second feature of this approach is that the latent interaction term is not allowed to correlate with the main effect latent variables. Because the indicators of the interaction term have been orthogonalized with respect to the main effect latent variables, the covariance matrix to be analyzed would contain covariances of precisely zero for the 54 possible relationships between the six main effect indicators and the nine interaction indicators. Appendix B contains the LISREL syntax for applying the orthogonalizing approach to the model in Figure 2. Appendix C contains the Mplus syntax for estimating each of the alternative models.

As shown in the first column of Table 3, the results of the latent variable interaction model showed a pattern of findings that are fully consistent with those from the OLS regressions. The only differences are that the magnitudes of the regression coefficients relating Positive Affect to Agency, Causes, and their interaction are larger, and their accompanying significance values smaller than the manifest variable representation shown in Table 2. Such an effect would
be expected from using multiple indicators of latent constructs to estimate structural relations disattenuated for measurement error. Also noteworthy is that the fit of the model containing the interaction term is well within acceptable ranges, a condition that is not often found with other techniques for representing latent variable interactions.

The model in Figure 2 was then re-parameterized based on the Marsh et al. (2004) unconstrained approach, which will be referred to as the “mean centered” approach, and the Klein and Moosbrugger (2000) LMS approach. Model results are also reported in Table 3. Note that, according to all reported fit indices, the orthogonalized approach out-performs the mean centered approach. Also note that, in terms of the primary model parameters (i.e. latent regressions and correlations), the orthogonalized approach results in comparable parameter estimates and t-ratios. Most traditional SEM fit indices are not available in Mplus when implementing the LMS approach to latent interactions, with the exception of the Akaike Information Criterion (AIC: Akaike, 1974) and Bayesian Information Criterion (BIC: Schwartz, 1978). Both AIC and BIC are information indices that can be used when comparing non-nested models – smaller values indicate better models both in terms of model fit and model parsimony. According to both the AIC and BIC, the LMS approach is preferable to both the mean-centered approach and the orthogonalized approach, however, all three approaches result in comparable latent parameter estimates and identical inferences.

Thus, the orthogonalized approach results in nearly identical parameter estimates as does the unconstrained mean-centered approach of Marsh et al. (2004) and the LMS approach (Klein & Moosbrugger, 2000). Although the LMS approach does result in lower AIC and BIC values, indicating a more parsimonious model, it achieves this deflated fit because the interaction construct is not estimated directly like the other two approaches and therefore far fewer
parameters are estimated. While both the mean-centered and orthogonalized approaches resulted in excellent model fit, the orthogonalized approach resulted in somewhat better fit, which is due to the complete orthogonality derived from residual centering that mean centering only approximates.

*Simulation Evidence of the Comparability of the Three Approaches*

While the three approaches for estimating latent variable interactions reported in Table 3 produced comparable results, there was not a consistent pattern as to the variability between approaches. For instance, the LMS approach produced a larger estimate for the Agency effect with a smaller t-ratio than both the orthogonalized or mean-centered approaches and a smaller estimate for the interaction effect but a larger t-ratio than the mean-centered approach. As a simple example of the efficacy of the orthogonalizing approach, we again compared three approaches to estimating latent variable interactions: the orthogonalizing approach described in this paper, the mean-centered approach described in Marsh et al. (2004), and the LMS method implemented in Mplus version 3. The orthogonalizing and mean-centered approaches both utilized all possible cross-product indicators to infer the latent constructs.

*Population model.* Mplus version 3.12 was used to generate 1000 replications, each with a sample size of 1500 participants. The sample size was chosen to reflect the actual sample size of the example data set used to generate the results reported in Table 3. The population (simulated) data was created to mimic the parameters reported in Marsh et al. (2004) but using the random slope parameterization implemented in Mplus (see Muthén & Asparouhov, 2003) with

\[
\begin{align*}
\eta_i &= 0.4\xi_{1i} + 0.4\xi_{2i} + s_i \xi_{1i} + \varepsilon_i \\
 s_i &= 0 + 0.2\xi_{2i} + 0
\end{align*}
\]

(14)

where \(s_i\) is a latent variable that only contributes one additional parameter, \(\beta_3 = .2\). The latent
exogenous constructs, $\xi_1$ and $\xi_2$, were standard normal variables, and the correlation coefficient between $\xi_1$ and $\xi_2$ was set at .3. Three indicators were used for each of the latent variables such that $y_1, y_2, y_3; x_1, x_2, x_3; \text{and } z_1, z_2, z_3$ were indicators of $\eta, \xi_1,$ and $\xi_2$ respectively. Construct identification was achieved by setting the latent variances to 1.0, and the loadings relating each indicator to its latent variable were all 0.70 in the population-generating model.

Table 4 outlines the results of the small simulation study. All replications resulted in proper solutions for all three methods. Across all 1000 replications, all factor loadings for $\eta, \xi_1, \text{and } \xi_2$ were estimated to average 0.70 with a standard error of 0.02 for all three approaches. Factor loadings for the interaction construct ($\xi_1 \xi_2$) under the mean-centered and orthogonalizing approaches were estimated to average 0.51 with a standard error of 0.02. The LMS approach does not require indicators, thus factor loadings for the interaction construct ($\xi_1 \xi_2$) were not estimated. For both the mean-centered and orthogonalized approaches, 76 free parameters were estimated: 18 factor loadings, 18 residual variances, 18 residual correlations, 3 latent regression coefficients, 1 latent correlation, and 18 intercepts. The LMS approach resulted in only 31 free parameters: 9 factor loadings, 9 residual variances, 3 latent regression coefficients, 1 latent correlation, and 9 intercepts.

Model fit. Chi-square, CFI, and RMSEA model fit information was only available for the mean-centered and orthogonalized approaches. With the same number of free parameters between both approaches and across 1000 replications, orthogonalizing resulted in a smaller average chi-square value (by a ratio of 2.25) and smaller standard deviation. Average CFI was equivalent to 2 decimal places between both approaches, and the average RMSEA differed by .008 when measured to three decimal places. As would be expected due to the significantly smaller number of free parameters estimated in the LMS approach, AIC and BIC values were
substantially smaller for the LMS approach versus both the mean centered and orthogonalized approach, but orthogonalizing resulted in lower AIC and BIC values than did mean centering.

*Latent coefficients.* All three approaches resulted in negligibly biased parameter estimates for both the effect of $\xi_1$ and $\xi_2$ with percent bias for $\xi_1$ ranging from 0.300% to 0.400% and percent bias for $\xi_2$ ranging from 0.025% to 0.125%. The latent correlation between $\xi_1$ and $\xi_2$ was estimated with negligible bias across all three approaches as well, ranging from 0.2% to 0.3%.

The primary difference between the three approaches involved the interaction effect. The LMS approach, which was also the generating model, produced only 0.05% bias when estimating the interaction effect, while both the orthogonalized and mean-centered approaches resulted in 3.9% and 4.2% bias, respectively. Although this bias seems to be a large discrepancy relative to other percent bias estimates, the parameter estimates differed only in the second decimal place between the mean-centered or orthogonalized approaches and the LMS approach, as shown in Table 4.

*Standard errors.* Also reported in Table 4, average standard errors for the latent parameters, both the effects of the latent constructs $\xi_1$, $\xi_2$, and $\xi_1\xi_2$ on $\eta$ and the correlation between $\xi_1$ and $\xi_2$, were very consistent across the three procedures, deviating only in the third decimal place. However, these values, when compared to the empirically obtained standard deviation of the sampling distribution of mean estimates (i.e., the population value for the standard error), tended to be biased. All three procedures resulted in negatively biased standard errors for the effect of $\xi_1$ and $\xi_1\xi_2$, and both the mean-centered and orthogonalized approaches resulted in negatively biased estimates for the effect of $\xi_2$. All three procedures tended to over-estimate the standard error for the latent correlation between $\xi_1$ and $\xi_2$. In general, the LMS approach resulted in a smaller percent bias than either of the other two procedures, especially for
the interaction effect.

Discussion

As mentioned, the goals of this short report were two-fold. First, we briefly highlighted some of the merits of residual centering interaction and powered terms in the standard regression context. Second, we outlined the implications of the orthogonalizing procedure to represent latent variable interactions. In our view, the proposed method for representing latent variable interactions has potential merits. First, the latent variable interaction is derived from the observed covariation pattern among all possible indicators of the interaction. Second, no constraints on particular estimated parameters need to be placed. Third, no recalculation of parameters are required. Finally, this procedure can be implemented in any standard structural model software.

In the simulation portion of this report, we generated data based on the random slopes approach to interactions implemented in Mplus and, as might be expected, all analyses involving the LMS approach performed better than the mean-centered or orthogonalized approach; however, the practical differences were generally trivial. However, there is a significant limitation in implementing the LMS approach – it requires either the use of (a) the specific Mplus software package, (b) a specialized software developed by the originator of the approach, or (c) specialized software developed by the researcher. Prior to Marsh et al. (2004), the only generally accessible procedures involved complex nonlinear constraints such as those described in Algina and Moulder (2001), Jaccard and Wan (1995), Jöreskog and Yang (1996), Schumacker and Marcoulides (1998), and Wall and Amemiya (2001). Like our orthogonalizing method, Marsh et al. proposed a relatively easy approach involving a simple mean-centering of the indicators prior to forming the cross products and not implementing the nonlinear constraints. The Marsh et al. approach, referred to here as a mean-centered approach, performed well relative
to the comparison procedures, which included an evolution of the LMS approach termed the QML approach (Klein & Muthén, 2002). We compared this simpler mean centered unconstrained approach with the LMS approach and our proposed orthogonalizing approach. We found that, not only did our orthogonalizing approach perform in a comparable manner to the comparison approaches, our approach performed somewhat better than the mean-centered approach.

Our approach, like most others, does suffer from the limitation that the standard errors, and thus, significance levels, of the parameters may be biased. Some of the other approaches, in fact, fail to give standard errors (Kenny & Judd, 1986) or, because of the significant number of constraints of estimates of the various parameters, give standard errors that are quite inflated (Jöreskog & Yang, 1993). However, the regression residuals that are used to estimate latent variable interactions from our orthogonalizing procedure are generally fairly normally distributed, thus the standard ML estimator likely provides a reasonably robust estimate of standard errors and significance. Although this assertion should be tested more thoroughly in future work, the finding that the observed standard errors were consistent with our expectations in the present example is quite encouraging. Bootstrap and robust estimation procedures could also be employed to estimate more precisely the standard errors of the parameter estimates associated with the latent variable interaction construct and their accompanying significance levels.

Future work is clearly needed to examine the robustness of the standard errors under more diverse conditions. Moreover, a direct Monte Carlo comparison of all the extant methods needs to be conducted in order to contrast the strengths and weaknesses of all the various approaches (including alternative parameterizations; Lubinski & Humphreys, 1990) across
different conditions, such as sample size, strength of effects, non-normality, reliability of the main effect indicators, and so on. While there have been several recent efforts on this front (e.g. Lee, Song, & Poon, 2004; Marsh et al., 2004; Moulder & Algina, 2002), none have been sufficiently comprehensive to resolve this debate.

In summary, we have proposed a new technique with which to represent latent variable interactions in SEM that is a relatively straightforward extension of orthogonalizing in regression frameworks (e.g., Lance, 1988). Moreover, orthogonalizing technique has the potential to be extended and applied to represent other non-linear constructs such as quadratic curvatures. In our view, the technique proposed here is a potential alternative method that a researcher can choose to implement. It is technically not highly demanding to specify, has direct practical interpretation of parameter estimates, and its behavior in terms of model fit and estimated standard errors in the current example appears to be very reasonable.
Footnotes

1In principle, orthogonalizing is not limited to use on variables that are mathematical manipulations of another variable. The technique can be readily applied on substantively derived variables. For example, Little, Brauner, Jones, Noch, and Hawley (2003) used orthogonalizing to separate two sources of variance from a multidimensional variable.
References


Muthén & Muthén.


Appendix A

SAS Code Used to Residual Center the Interaction Terms and Generate the Data for the Orthogonalized Latent Variable Interaction Factor

```sas
DATA RC;
Set raw;
  x1z1 = x1*z1;
  x1z2 = x1*z2;
  x1z3 = x1*z3;
  x2z1 = x2*z1;
  x2z2 = x2*z2;
  x2z3 = x2*z3;
  x3z1 = x3*z1;
  x3z2 = x3*z2;
  x3z3 = x3*z3;
RUN;

PROC REG noprint data=RC;
  Model x1z1 = x1 x2 x3 z1 z2 z3;
  Output out=RC r=o_x1z1; RUN;
PROC REG noprint data=RC;
  Model x1z2 = x1 x2 x3 z1 z2 z3;
  Output out=RC r=o_x1z2; RUN;
PROC REG noprint data=RC;
  Model x1z3 = x1 x2 x3 z1 z2 z3;
  Output out=RC r=o_x1z3; RUN;
PROC REG noprint data=RC;
  Model x2z1 = x1 x2 x3 z1 z2 z3;
  Output out=RC r=o_x2z1; RUN;
PROC REG noprint data=RC;
  Model x2z2 = x1 x2 x3 z1 z2 z3;
  Output out=RC r=o_x2z2; RUN;
PROC REG noprint data=RC;
  Model x2z3 = x1 x2 x3 z1 z2 z3;
  Output out=RC r=o_x2z3; RUN;
PROC REG noprint data=RC;
  Model x3z1 = x1 x2 x3 z1 z2 z3;
  Output out=RC r=o_x3z1; RUN;
PROC REG noprint data=RC;
  Model x3z2 = x1 x2 x3 z1 z2 z3;
  Output out=RC r=o_x3z2; RUN;
PROC REG noprint data=RC;
  Model x3z3 = x1 x2 x3 z1 z2 z3;
  Output out=RC r=o_x3z3; RUN;
```

Orthogonalizing Approach

Note. This model was specified using all ‘Y’-side matrices because the distinction between ‘X’ and ‘Y’ is simply a theoretical distinction that does not impact the estimates if the variables are specified in LISREL as ‘Y’ variables. Conceptually, the first three latent variables can be thought of as exogenous and are treated as such because no other variables predict them even though they are listed on the ‘Y’-side. However, it is just as easy to consider the first three latent variables as endogenous to some unknown exogenous factors, and therefore, they can be specified on the ‘Y’ with the idea that future research may identify and include endogenous factors or potential covariate effects as ‘X’-side, exogenous factors.

DA NI=18 NO=1503 MA=CM

LA
  x1 x2 x3 z1 z2 z3
  o_x1z1 o_x1z2 o_x1z3 o_x2z1 o_x2z2 o_x2z3 o_x3z1 o_x3z2 o_x3z3
  y1 y2 y3
me=means.dat
sd=stdev.dat
km=matrix.dat
MO NY=18 NE=4 LY=FU,FI PS=SY,FI BE=FU,FI TE=SY,FI
FR LY(1,1) LY(2,1) LY(3,1)
FR LY(4,2) LY(5,2) LY(6,2)
FR LY(7,3) LY(8,3) LY(9,3) LY(10,3) LY(11,3) LY(12,3) LY(13,3) LY(14,3) LY(15,3)
FR LY(16,4) LY(17,4) LY(18,4)
VA 1 PS(1,1) PS(2,2) PS(3,3) PS(4,4) !identifying and establishing scale
FR PS(2,1)
FR BE(4,1) BE(4,2) BE(4,3)
FR TE(1,1) TE(2,2) TE(3,3) TE(4,4) TE(5,5) TE(6,6) TE(7,7)
FR TE(8,8) TE(9,9) TE(10,10) TE(11,11) TE(12,12) te(13,13)
FR TE(14,14) TE(15,15) TE(16,16) TE(17,17) TE(18,18)
FR TE(8,7) TE(9,7) TE(9,8)
FR TE(11,10) TE(12,10) TE(12,11)
The basic syntax for estimating a latent variable interaction using either the orthogonalizing approach proposed in this paper or the mean centered approach used for comparison purposes is identical. The difference between the two procedures lies in the data that is used for input. The data for the orthogonalized approach would include product indicator variables for the interaction factor that have been residual centered as described in Appendix A. The data for the mean centered approach would include interaction indicators that have been mean-centered.

**LMS Approach**

As of this writing, it is not possible to implement the LMS approach using LISREL.
Appendix C

Mplus 3.12 Syntax for Estimating Latent Variable Interaction Terms

Orthogonalizing Approach

The following Mplus syntax examples begin with the ANALYSIS command. It is assumed that the reader is familiar with specifying the TITLE, DATA, and VARIABLE commands in Mplus.

ANALYSIS: TYPE = MEANSTRUCTURE;
ESTIMATOR = ML;
MODEL:
X BY x1* x2* x3*;
Z BY z1* z2* z3*;
Y BY y1* y2* y3*;
INT BY x1z1* x1z2* x1z3* x2z1* x2z2* x2z3* x3z1* x3z2* x3z3*;
X@1;Z@1;Y@1;INT@1;

X WITH Z*;
X WITH INT@0;
Z WITH INT@0;
Y ON X Z INT;

x1z1 WITH x1z2* x1z3* x2z1* x2z2@0 x2z3@0 x3z1* x3z2@0 x3z3@0;
x1z2 WITH x1z3* x2z1@0 x2z2* x2z3@0 x3z1@0 x3z2@0 x3z3@0;
x1z3 WITH x2z1@0 x2z2@0 x2z3* x3z1@0 x3z2@0 x3z3*;
x2z1 WITH x2z2* x2z3* x3z1* x3z2@0 x3z3@0;
x2z2 WITH x2z3* x3z1@0 x3z2* x3z3@0;
x2z3 WITH x3z1@0 x3z2@0 x3z3*;
x3z1 WITH x3z2* x3z3*;
x3z2 WITH x3z3*;

[y1 y2 y3 x1 x2 x3 z1 z2 z3 x1z1 x1z2 x1z3 x2z1 x2z2 x2z3 x3z1 x3z2 x3z3];

Mean Centered Approach

As was the case in Appendix B, the only functional difference between implementing the orthogonalizing approach and the mean centered approach is in the input data file(s). The orthogonalizing approach requires residual centered product indicators for the interaction term, while the mean centered approach requires mean centered product indicators.
LMS Approach

The following Mplus syntax examples begin with the ANALYSIS command. It is again assumed that the reader is familiar with specifying the TITLE, DATA, and VARIABLE commands in Mplus.

ANALYSIS:   TYPE = RANDOM;
            ALGORITHM = INTEGRATION;

MODEL:     X BY x1* x2* x3*;
            Z BY z1* z2* z3*;
            Y BY y1* y2* y3*;

            XZ | X XWITH Z;

            Y ON X Z XZ;

            X@1; Y@1; Z@1;
Appendix D

Mplus 3.12 Syntax for Monte Carlo Data Generation

Data Generation

TITLE: Data generation syntax.

MONTECARLO:
NAMES ARE y1 y2 y3 x1 x2 x3 z1 z2 z3;
NOBSERVATIONS = 1500;
NREPS = 1000;
SEED = 53487;
REPSAVE = ALL;
SAVE = C:\data*.txt;

ANALYSIS: TYPE = RANDOM;
ALGORITHM = INTEGRATION;

MODEL MONTECARLO:
[x1-x3*3 z1-z3*3 y1-y3*3];
x1-x3*.3; z1-z3*.3; y1-y3*.3;
X BY x1*.7 x2*.7 x3*.7;
Z BY z1*.7 z2*.7 z3*.7;
Y BY y1*.7 y2*.7 y3*.7;
X@1;  Y@1;  Z@1;

XZ | X XWITH Z;

Y ON X*.4 Z*.4 XZ*.2;

Z WITH X*.3;

Data Manipulation

A SAS macro was then used to modify each dataset to match the input requirements for the particular comparison approach. Thus, two additional versions of each of the 1000 replications were created. The original simulated data was used for the LMS approach. A second set of replications were created where all X and Z variables were mean centered prior to forming all possible product indicators for the mean centered approach. Finally, a third set of replications
were created for the orthogonalizing approach where all possible X-Z product indicators were created and then residual centered as demonstrated in Appendix A.

**Monte Carlo Data Analysis**

Once replication data were simulated and manipulated, the following DATA command was used in conjunction with the Mplus syntax presented in Appendix C for automated analysis of the multiple data sets and summarization of the replication results as detailed in the Mplus manual (Muthén & Muthén, 1998-2004).

```
DATA:  FILE IS C:\datalist.txt;
       TYPE = MONTECARLO;
```
Author Note

Author contribution by the first two authors was equivalent. James Bovaird is now at the University of Nebraska-Lincoln. Correspondence regarding this article should be addressed to Todd Little (yhat@ku.edu) at Schiefelbusch Institute for Life Span Studies, University of Kansas, 1000 Sunnyside Avenue, Lawrence, KS 66045-7555, or to James Bovaird (jbovaird2@unl.edu) at Department of Educational Psychology, University of Nebraska-Lincoln, 36 Teachers College Hall, Lincoln, NE 68588-0345.

This work was supported in part by grants from the NIH to the University of Kansas through the Mental Retardation and Developmental Disabilities Research Center (5 P30 HD002528), the Center for Biobehavioral Neurosciences in Communication Disorders (5 P30 DC005803), a new faculty grant (NFGRF 2301779) from the University of Kansas to the first author, and an Institutional National Research Service Award (5 T32 HD07525-04) to the second author.
Footnotes

1In principle, orthogonalizing is not limited to use on variables that are mathematical manipulations of another variable. The technique can be readily applied on substantively derived variables. For example, Little, Brauner, Jones, Noch, and Hawley (2003) used orthogonalizing to separate two sources of variance from a multidimensional variable.
Table 1

*Simple Statistics and Pearson Correlations among the Variables Used in the Regressions*

<table>
<thead>
<tr>
<th></th>
<th>Positive Affect</th>
<th>Agency</th>
<th>Causes</th>
<th>INT_U</th>
<th>INT_MC</th>
<th>INT_O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive Affect</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agency</td>
<td>0.22</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Causes</td>
<td>-0.01</td>
<td>-0.24</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>INT_U</td>
<td>0.10</td>
<td>0.37</td>
<td>0.80</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>INT_MC</td>
<td>-0.10</td>
<td>0.05</td>
<td>-0.06</td>
<td>0.16</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>INT_O</td>
<td>-0.11</td>
<td>0.00</td>
<td>0.00</td>
<td>0.18</td>
<td>0.99</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Positive Affect</th>
<th>Agency</th>
<th>Causes</th>
<th>INT_U</th>
<th>INT_MC</th>
<th>INT_O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>3.03</td>
<td>3.12</td>
<td>2.02</td>
<td>5.35</td>
<td>0.06</td>
<td>0.00</td>
</tr>
<tr>
<td>SD</td>
<td>0.68</td>
<td>0.54</td>
<td>0.52</td>
<td>1.68</td>
<td>0.30</td>
<td>0.30</td>
</tr>
</tbody>
</table>

*Note:* INT_U is the straight product of the two first-order effects (Agency and Causes); INT_MC is the interaction term created after mean-centering the two first-order effects; INT_O is the residual centered interaction term.
Table 2

Comparison of Results from Three Standard Regression Approaches

<table>
<thead>
<tr>
<th>Predictor</th>
<th>$R_{xx}$</th>
<th>Uncentered</th>
<th>Mean Centered</th>
<th>Residual Centered</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Beta</td>
<td>$t$</td>
<td>Beta</td>
</tr>
<tr>
<td>First-order effects Only Regressions$^1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agency</td>
<td>.66</td>
<td>.23</td>
<td>8.79</td>
<td>.23</td>
</tr>
<tr>
<td>Causes</td>
<td>.74</td>
<td>.05</td>
<td>1.77</td>
<td>.05</td>
</tr>
<tr>
<td>Adding the Respective Interaction Terms</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agency</td>
<td>.66</td>
<td>.59</td>
<td>6.81</td>
<td>.23</td>
</tr>
<tr>
<td>Causes</td>
<td>.74</td>
<td>.62</td>
<td>4.63</td>
<td>.04</td>
</tr>
<tr>
<td>Interaction</td>
<td>.76</td>
<td>-.62</td>
<td>-4.37</td>
<td>-.11</td>
</tr>
</tbody>
</table>

Note. Reliability of Positive Affect is .92

$^1$ The first-order effects only regressions are represented three times only for pedagogic reasons. The regression was only done one time as it is the basis for each subsequent interaction test.
Table 3

*Comparison of Results from Three Latent Variable Interaction Approaches*

<table>
<thead>
<tr>
<th></th>
<th>Orthogonalized</th>
<th>Mean Centered</th>
<th>LMS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model Fit</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi^2_{(113)}$</td>
<td>174.038</td>
<td>445.61</td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>CFI</td>
<td>0.994</td>
<td>0.966</td>
<td></td>
</tr>
<tr>
<td>TLI</td>
<td>0.991</td>
<td>0.954</td>
<td></td>
</tr>
<tr>
<td>AIC</td>
<td>38163.29</td>
<td>38434.89</td>
<td>23109.33</td>
</tr>
<tr>
<td>BIC</td>
<td>38471.45</td>
<td>38743.05</td>
<td>23274.04</td>
</tr>
<tr>
<td>RMSEA</td>
<td>0.019</td>
<td>0.044</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>b</th>
<th>t</th>
<th>b</th>
<th>t</th>
<th>b</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Latent Regressions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agency</td>
<td>0.291</td>
<td>7.877</td>
<td>0.296</td>
<td>7.981</td>
<td>0.304</td>
<td>6.822</td>
</tr>
<tr>
<td>Causes</td>
<td>0.075</td>
<td>2.158</td>
<td>0.071</td>
<td>2.046</td>
<td>0.070</td>
<td>1.908</td>
</tr>
<tr>
<td>Agency*Causes</td>
<td>-0.182</td>
<td>-4.327</td>
<td>-0.173</td>
<td>-4.08</td>
<td>-0.151</td>
<td>-4.275</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>r</th>
<th>t</th>
<th>r</th>
<th>t</th>
<th>r</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Latent Correlations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agency w/ Causes</td>
<td>-0.289</td>
<td>-8.486</td>
<td>-0.289</td>
<td>-8.494</td>
<td>-0.294</td>
<td>-7.432</td>
</tr>
</tbody>
</table>
Table 4  
*Comparison of Simulation Results from Three Latent Variable Interaction Approaches*

<table>
<thead>
<tr>
<th>Model Fit</th>
<th>Orthogonalized Mean</th>
<th>SD</th>
<th>% Bias</th>
<th>Mean Centered Mean</th>
<th>SD</th>
<th>% Bias</th>
<th>LMS Mean</th>
<th>SD</th>
<th>% Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>χ²(113)</td>
<td>56.26</td>
<td>11.00</td>
<td>--</td>
<td>126.81</td>
<td>19.54</td>
<td>--</td>
<td>n/a</td>
<td>n/a</td>
<td>--</td>
</tr>
<tr>
<td>CFI</td>
<td>1.00</td>
<td>0.00</td>
<td>--</td>
<td>1.00</td>
<td>0.00</td>
<td>--</td>
<td>n/a</td>
<td>n/a</td>
<td>--</td>
</tr>
<tr>
<td>AIC</td>
<td>51841.8</td>
<td>594.5</td>
<td>--</td>
<td>51912.6</td>
<td>594.6</td>
<td>--</td>
<td>30070.5</td>
<td>167.7</td>
<td>--</td>
</tr>
<tr>
<td>BIC</td>
<td>52245.6</td>
<td>594.5</td>
<td>--</td>
<td>52316.2</td>
<td>594.6</td>
<td>--</td>
<td>30235.3</td>
<td>167.7</td>
<td>--</td>
</tr>
<tr>
<td>RMSEA</td>
<td>0.000</td>
<td>0.000</td>
<td>--</td>
<td>0.008</td>
<td>0.006</td>
<td>--</td>
<td>n/a</td>
<td>n/a</td>
<td>--</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Latent Regressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>ξ₁</td>
</tr>
<tr>
<td>ξ₂</td>
</tr>
<tr>
<td>ξ₁*ξ₂</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Latent Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>ξ₁ with ξ₂</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Standard Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>ξ₁</td>
</tr>
<tr>
<td>ξ₂</td>
</tr>
<tr>
<td>ξ₁*ξ₂</td>
</tr>
<tr>
<td>ξ₁ with ξ₂</td>
</tr>
</tbody>
</table>
Figure Captions

*Figure 1.* The nature of the interaction. *Note.* The slope varies from .23 to -.45 as a function of Unknown Causes

*Figure 2.* A latent variable interaction using orthogonalized indicators. *Note.* The t-value is listed in the parentheses next to the standardized estimate.
Figure 2 - TOP

Fit Information:
X²(113) = 168.9, p = .0005,
RMSEA = .018, NNFI = .997

Orthogonalized Interaction

Agency Ability

Unknown Causes

Positive Affect (R² = .10)

1

2

3

.29 (7.83)

- .28 (8.28)

.07 (2.10)

4

5

6

- .19 (4.41)